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Stress energy tensors in the theories of direct interparticle action

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Abstract. It is shown that the canonical tensor and not the Frenkel tensor is the correct tensor to describe the gravitational influence of direct particle fields. This corrects an earlier calculation which had resulted in favour of the Frenkel tensor. The forms of the two tensors are examined in cosmological models with perfect future absorbers and imperfect past absorbers, and it is shown that only the canonical tensor is physically reasonable.

1. Introduction

Recent investigations (Hogarth 1962, Hoyle and Narlikar 1963, 1969, 1971, Roe 1969, Davies 1971) have shown that in suitable cosmological models it is possible to describe electromagnetism in terms of direct interactions between charged particles. This picture does away with the notion of 'field' as an independent entity, a notion which plays such an important part in maxwellian electrodynamics. Instead we have the so-called 'direct particle fields' (ie fields which are defined in terms of charged particles). In this paper we shall refer to this alternative theory of electromagnetism as the direct particle theory.

Any theory proposed as an alternative to Maxwell's theory must meet at least two requirements. First, it must successfully account for all the electrodynamic phenomena associated with charged leptons at the quantum as well as the classical level. Secondly, it must provide a consistent picture of the gravitational influence exerted by electromagnetism through Einstein's equations of gravitation. The first requirement has been adequately met as shown in the work referred to above. What about the second requirement? In the Maxwell theory, the fields define a stress energy tensor, which acts as a source of Einstein's equations. In the absence of fields how does the direct particle theory deal with this problem?

In this pioneering work on action at a distance, Wheeler and Feynman (1949) had arrived at two possible forms of the stress energy tensors, the Frenkel tensor and the canonical tensor, each of which served equally well to describe the exchange of energy and momentum between charged particles. However, this discussion was confined to electrodynamics alone, and the authors concluded: 'From the standpoint of pure electrodynamics it is not possible to choose between the two tensors. The difference is of course significant for the general theory of relativity, where energy has associated with it a gravitational mass. So far we have not attempted to discriminate between the two possibilities by way of this higher standard.'

Such a discrimination was attempted by Hoyle and Narlikar (1964, to be referred to as I) and their conclusion was that the 'correct' tensor is the Frenkel tensor. However, the computation leading to this conclusion missed one subtle point which in fact alters the result. The purpose of this paper is to redo the computation which now results in favour of the canonical tensor. As we shall see, this result not only applies to electrodynamics but also to other direct particle theories.

2. Action at a distance electrodynamics

We begin with a brief description of electrodynamics of direct interparticle action in riemannian space–time. (For details, see Hoyle and Narlikar 1963.) The starting point is the Fokker action

$$S = -\sum_a \int m_a \, \mathrm{d}a - \sum_{a<b}\sum 4\pi e_a e_b \iint \bar{G}_{i_\mathrm{A} i_\mathrm{B}} \, \mathrm{d}a^{i_\mathrm{A}} \, \mathrm{d}b^{i_\mathrm{B}}, \tag{1}$$

where $a, b, \ldots$ are the charged particles, e_a, m_a being the charge and mass of the ath particle. $\mathrm{d}a^{i_\mathrm{A}}$ represents the coordinate differentials at a typical point A on the world line of a, and $\mathrm{d}a$ is the element of proper time at A. The propagator $\bar{G}_{i_\mathrm{A} i_\mathrm{B}}$ represents the interaction between typical points A on the world line of a and B on the world line of b, and it acts like a vector at either point. It satisfies the symmetry condition

$$\bar{G}_{i_\mathrm{A} i_\mathrm{B}} = \bar{G}_{i_\mathrm{B} i_\mathrm{A}}. \tag{2}$$

If we keep A fixed and replace B by a variable point X, $\bar{G}_{i_\mathrm{X} i_\mathrm{A}}$ satisfies the wave equation

$$\Box \bar{G}_{i_\mathrm{X} i_\mathrm{A}} + R_{i_\mathrm{X}}{}^{l_\mathrm{X}} \bar{G}_{l_\mathrm{X} i_\mathrm{A}} = \delta_4(\mathrm{X}, \mathrm{A})(-\bar{g}(\mathrm{X}, \mathrm{A}))^{-1/2} \bar{g}_{i_\mathrm{X} i_\mathrm{A}} \tag{3}$$

in which the $\Box$ is the wave operator with respect to X, $R_i^{\,l}$ is the Ricci tensor, $\delta_4(\mathrm{X}, \mathrm{A})$ is the four-dimensional delta function, $\bar{g}_{i_\mathrm{X} i_\mathrm{A}}$ is the parallel propagator defined by Synge (1960) and $\bar{g}(\mathrm{X}, \mathrm{A})$ its determinant. We also have the relation

$$\bar{G}^{i_\mathrm{X}}{}_{k_\mathrm{A}; i_\mathrm{X}} = -\bar{G}(\mathrm{X}, \mathrm{A})_{;k_\mathrm{A}}, \tag{4}$$

where $\bar{G}(\mathrm{X}, \mathrm{A})$ is the symmetric solution of the scalar wave equation

$$\Box \bar{G}(\mathrm{X}, \mathrm{A}) = \delta_4(\mathrm{X}, \mathrm{A})(-\bar{g}(\mathrm{X}, \mathrm{A}))^{-1/2}. \tag{5}$$

The propagators $\bar{G}_{i_\mathrm{X} i_\mathrm{A}}$ and $\bar{G}(\mathrm{X}, \mathrm{A})$ have delta function supports on the null cone from A and also support inside the null cone (DeWitt and Brehme 1960). In flat space, when the space–time metric g_{ik} becomes the Minkowski metric $\eta_{ik} = \mathrm{diag}(-1, -1, -1, 1)$ we have

$$\bar{G}_{i_\mathrm{X} k_\mathrm{A}} = \frac{1}{4\pi} \delta(s_{\mathrm{XA}}^2) \eta_{ik}, \qquad \bar{G}(\mathrm{X}, \mathrm{A}) = \frac{1}{4\pi} \delta(s_{\mathrm{XA}}^2). \tag{6}$$

The action (1) describes the action at a distance between pairs of charged particles. Because of the symmetry (2) the charges interact equally via advanced and retarded interactions (see figure 1). These can be described by 'direct particle' potentials and fields which are defined as follows:

$$A_{i_\mathrm{X}}^{(a)} = 4\pi e_a \int \bar{G}_{i_\mathrm{X} i_\mathrm{A}} \, \mathrm{d}a^{i_\mathrm{A}}, \qquad F_{i_\mathrm{X} k_\mathrm{X}}^{(a)} = A_{k_\mathrm{X}; i_\mathrm{X}}^{(a)} - A_{i_\mathrm{X}; k_\mathrm{X}}^{(a)}. \tag{7}$$

Note that $A_{i_\mathrm{X}}^{(a)}$ and $F_{i_\mathrm{X} k_\mathrm{X}}^{(a)}$ are defined in terms of particle a; they have no existence independent of the particle. They satisfy the Maxwell equations and the Lorentz gauge condition identically.

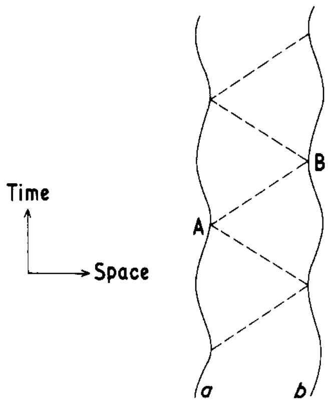

Figure 1. Delayed action at a distance between charged particles a and b operates via retarded and advanced effects.

At first sight the perfect time symmetry of this description poses problems. Instead of the retarded fields observed in nature, we seem to have a 'half and half' combination of retarded and advanced fields. Wheeler and Feynman (1945) resolved this difficulty by appealing to the time asymmetry in thermodynamics in a static universe. However, a more natural resolution came when proper account was taken of cosmology (Hogarth 1962, Hoyle and Narlikar 1963). We shall discuss the cosmological implications briefly towards the end.

3. The electromagnetic stress energy tensor

We now turn to the central theme of this paper: that of determining the stress energy tensor of electromagnetic theory as described in the last section. It is well known that the general theory of relativity provides a prescription for determining the stress energy tensor for any field theory described in riemannian space. Suppose Φ is a field of arbitrary tensorial rank, described by an action

$$\Lambda = \int L[\Phi]\surd(-g)\,\mathrm{d}^4x \tag{8}$$

where $L[\Phi]$ is the field lagrangian which is a scalar built of Φ and its derivatives. If we now perform a variation of the metric the change in Λ is expressed in the form

$$\delta\Lambda = \int -\tfrac{1}{2}T^{ik}[\Phi]\delta g_{ik}\surd(-g)\,\mathrm{d}^4x, \tag{9}$$

then $T^{ik}[\Phi]$ is identified as the stress energy tensor of Φ. For the Maxwell field F_{ik} we have

$$L[F] = -\frac{1}{16\pi}F_{mn}F^{mn} \tag{10}$$

and (9) leads to

$$T^{ik}[F] = \frac{1}{4\pi}(\tfrac{1}{4}F^{lm}F_{lm}g^{ik} - F^{i}{}_{l}F^{kl}). \tag{11}$$

Can this prescription be used for direct particle theories? At first sight the answer appears to be in the negative. The direct particle theories do not have a term in the action depending only on fields, since fields do not exist as independent entities in such theories. Nevertheless, the action as a whole does depend on space–time geometry in a non-trivial way and a change of g_{ik} leads to a change of Λ even in a direct particle theory. For instance in the electromagnetic case, if we write

$$\Lambda = -\sum_{a<b}\sum 4\pi e_a e_b \iint \bar{G}_{i_A k_B}\, \mathrm{d}a^{i_A}\, \mathrm{d}b^{k_B}, \tag{12}$$

then $g_{ik} \to g_{ik} + \delta g_{ik}$ leads to $\bar{G}_{i_A k_B} \to \bar{G}_{i_A k_B} + \delta\bar{G}_{i_A k_B}$ and hence to $\Lambda \to \Lambda + \delta\Lambda$. We then define T^{ik} by

$$\delta\Lambda = \int -\tfrac{1}{2}T^{ik}\delta g_{ik}\sqrt{(-g)}\, \mathrm{d}^4x, \tag{13}$$

as in (9). To determine T^{ik} we therefore need to calculate $\delta\bar{G}_{i_A k_B}$.

To evaluate $\delta\bar{G}_{i_A k_B}$ we shall follow the procedure of I, although details will turn out to be different. We begin with definitions. Writing ordinary derivatives with commas, define

$$\mathscr{F}_{i_X k_X i_A} = \bar{G}_{k_X i_A; i_X} - \bar{G}_{i_X i_A; k_X}. \tag{14}$$

Then from (7) we get

$$F^{(a)}_{i_X k_X} = 4\pi e_a \int \mathscr{F}_{i_X k_X i_A}\, \mathrm{d}a^{i_A}. \tag{15}$$

It will be convenient for later work to define the advanced and retarded components of various propagators. $\bar{G}_{i_X k_A}$ has support inside and on the light cone from A or X. Considering A as the source point and X as the general point we write

$$\bar{G}_{i_X k_A} = \tfrac{1}{2}(G^{\text{ret}}_{i_X k_A} + G^{\text{adv}}_{i_X k_A}), \tag{16}$$

where $\frac{1}{2}G^{\text{ret}}_{i_X k_A}$ is that part of $\bar{G}_{i_X k_A}$ which lies on the future light cone of A. $\frac{1}{2}G^{\text{adv}}_{i_X k_A}$ is similarly the part of $\bar{G}_{i_X k_A}$ having support inside and on the past light cone of A. We then have

$$G^{\text{ret}}_{k_A i_X} = G^{\text{adv}}_{i_X k_A}, \qquad G^{\text{adv}}_{k_A i_X} = G^{\text{ret}}_{i_X k_A}. \tag{17}$$

In the same way we write

$$\mathscr{F}_{i_X k_X i_A} = \tfrac{1}{2}(\mathscr{F}^{\text{ret}}_{i_X k_X i_A} + \mathscr{F}^{\text{adv}}_{i_X k_X i_A}). \tag{18}$$

The advanced and retarded components of $\mathscr{F}_{i_X k_X i_A}$ can be related to the advanced and retarded direct particle fields of the last section:

$$F^{(a)}_{i_X k_X\,\text{adv}} = \int \mathscr{F}^{\text{adv}}_{i_X k_X i_A}\, \mathrm{d}a^{i_A}, \tag{19}$$

$$F^{(a)}_{i_X k_X\,\text{ret}} = \int \mathscr{F}^{\text{ret}}_{i_X k_X i_A}\, \mathrm{d}a^{i_A}. \tag{20}$$

With these definitions we now proceed to determine $\delta\bar{G}_{i_A k_B}$.

As in I we recast (3) in the form

$$(g^{il}g^{mk}\sqrt{(-g)}\mathscr{F}_{mli_A})_{,k} + \sqrt{(-g)}g^{il}\bar{G}^{k}{}_{i_A;kl} = \delta_4(\mathrm{X}, \mathrm{A})\bar{g}^{i}{}_{i_A} \tag{21}$$

where we have dropped the suffix X from the vector indices at X. This saves a lot of writing without causing ambiguity.

A variation of (21) under $g_{ik} \rightarrow g_{ik} + \delta g_{ik}$ gives therefore

$$(g^{il}g^{mk}\sqrt{(-g)}\delta\mathscr{F}_{mli_{\mathrm{A}}})_{,k} + \sqrt{(-g)}g^{il}\left(\frac{1}{\sqrt{(-g)}}(\sqrt{(-g)}g^{km}\delta\bar{G}_{mi_{\mathrm{A}}})_{,k}\right)_{,l} = Q^{i}{}_{i_{\mathrm{A}}} \quad (22)$$

where, after some mathematical reduction,

$$Q^{i}{}_{i_{\mathrm{A}}} = -[\delta(g^{il}g^{mk}\sqrt{(-g)})\mathscr{F}_{mli_{\mathrm{A}}}]_{,k} + \delta(\sqrt{(-g)}g^{il})\bar{G}(\mathrm{X},\mathrm{A})_{,i_{\mathrm{A}}l}$$
$$+\sqrt{(-g)}g^{il}\left[\delta\left(\frac{1}{\sqrt{(-g)}}\right)\sqrt{(-g)}\bar{G}(\mathrm{X},\mathrm{A})_{,i_{\mathrm{A}}} - \frac{1}{\sqrt{(-g)}}[\delta(\sqrt{(-g)}g^{km})\bar{G}_{mi_{\mathrm{A}}}]_{,k}\right]_{,l}. \quad (23)$$

In the above reduction we have used (4) in the second and third terms of $Q^{i}{}_{i_{\mathrm{A}}}$.

The wave operator in (22) is the same as that in (21). We can therefore use the Green functions of (21) to construct the solution of (22). If the variation of geometry is confined to a region V of the space–time, we can calculate $\delta\bar{G}_{i_{\mathrm{B}}i_{\mathrm{A}}}$ formally from (22) in the form

$$\delta\bar{G}_{i_{\mathrm{B}}i_{\mathrm{A}}} = \int_{\mathrm{V}} \bar{G}_{i_{\mathrm{B}}i}Q^{i}{}_{i_{\mathrm{A}}}\,\mathrm{d}^4x. \quad (24)$$

In (24) we have used $\bar{G}_{i_{\mathrm{B}}i}$ as the Green function mentioned above. This seems a natural step to take since all along we have been considering the symmetric Green function explicitly. Also (24) guarantees that $\delta\bar{G}_{i_{\mathrm{B}}i_{\mathrm{A}}} = \delta\bar{G}_{i_{\mathrm{A}}i_{\mathrm{B}}}$ which is an essential requirement here. This procedure, followed in I, leads to the Frenkel tensor

$$T^{ik} = \frac{1}{8\pi}\sum\sum_{a\neq b}(\tfrac{1}{2}g^{ik}F^{(a)mn}F^{(b)}_{mn} - F^{(a)il}F^{(b)k}{}_{l} - F^{(b)il}F^{(a)k}{}_{l}). \quad (25)$$

However, as we shall now see, this apparently reasonable procedure, is not correct. To see this we write (24) in an explicit form obtained after some straightforward use of the divergence theorem (cf I for details):

$$\delta\bar{G}_{i_{\mathrm{B}}i_{\mathrm{A}}} = -\tfrac{1}{2}\int_{\mathrm{V}} \delta(g^{il}g^{mk}\sqrt{(-g)})\mathscr{F}_{mli_{\mathrm{A}}}\mathscr{F}_{iki_{\mathrm{B}}}\,\mathrm{d}^4x$$
$$+\int_{\mathrm{V}} \delta(\sqrt{(-g)}g^{il})(\bar{G}_{i_{\mathrm{B}}i}\bar{G}(\mathrm{X},\mathrm{A})_{,i_{\mathrm{A}}l} + \bar{G}_{ii_{\mathrm{A}}}\bar{G}(\mathrm{B},\mathrm{X})_{,i_{\mathrm{B}}l})\,\mathrm{d}^4x$$
$$-\int_{\mathrm{V}} \delta(\sqrt{(-g)})\bar{G}(\mathrm{X},\mathrm{A})_{,i_{\mathrm{A}}}\bar{G}(\mathrm{B},\mathrm{X})_{,i_{\mathrm{B}}}\,\mathrm{d}^4x. \quad (26)$$

That $\delta\bar{G}_{i_{\mathrm{B}}i_{\mathrm{A}}} = \delta\bar{G}_{i_{\mathrm{A}}i_{\mathrm{B}}}$ is immediately obvious from (26) and from the symmetry properties of the Green functions involved. There is, however, another important property to reckon with. For, we know that $\bar{G}_{i_{\mathrm{B}}i_{\mathrm{A}}}$ is zero whenever B lies outside the light cone of A (ie, whenever $s^2_{\mathrm{AB}} < 0$). In a small variation of geometry the light cones are expected to change only slightly. So if $s^2_{\mathrm{AB}} < 0$ originally, we would expect $s^2_{\mathrm{AB}} < 0$ after the variation, unless $s^2_{\mathrm{AB}} \simeq 0$. We shall consider the situation illustrated in figure 2, where $s^2_{\mathrm{AB}} < 0$. The shaded region V is the one where $\delta g_{ik} \neq 0$ and X is a typical point of V such that $s^2_{\mathrm{AX}} > 0$, $s^2_{\mathrm{BX}} > 0$. Clearly, for such a point the integrands in (26) will not in general vanish. Hence, except in very special cases of δg_{ik} and V we will have $\delta\bar{G}_{i_{\mathrm{B}}i_{\mathrm{A}}} \neq 0$.

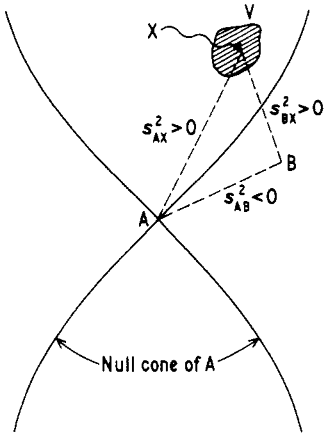

Figure 2. The point X in V lies inside the null cones of A and B while B lies outside the null cone of A. The correct solution for $\delta\bar{G}_{i_B i_A}$ should not include any variational contributions from such points as X. This is ensured by the use of (30).

The reason why this has happened is that the wrong Green function was used in the solution of (22). In general we can use any suitable linear combination of the advanced and retarded Green functions defined through (16) because these Green functions also satisfy the equation (3). In the correct solution the combination has to be adjusted in such a way to ensure the symmetry condition $\delta\bar{G}_{i_B i_A} = \delta\bar{G}_{i_A i_B}$ as well as the support condition that $\delta\bar{G}_{i_B i_A} = 0$ for $s^2_{AB} < 0$. It is not difficult to spot the correct combination and to verify that it is unique. We state the answer:

$$\delta\bar{G}_{i_B i_A} = \tfrac{1}{2}\int (G^{\text{ret}}_{i_B i} Q^{i\,\text{ret}}_{\;\;i_A} + G^{\text{adv}}_{i_B i} Q^{i\,\text{adv}}_{\;\;i_A})\,\mathrm{d}^4x, \tag{27}$$

where, as in (16) and (18), we write

$$Q^{i}_{\;\;i_A} = \tfrac{1}{2}(Q^{i\,\text{ret}}_{\;\;i_A} + Q^{i\,\text{adv}}_{\;\;i_A}). \tag{28}$$

This break up of the problem into retarded and advanced components is possible provided the topological properties of space are not so weird as to mix up the past and future light cones from any point.

The evaluation of (27) is straightforward if a little cumbersome. To illustrate the use of the definitions and the properties (16)–(18) in this calculation we will evaluate one term of the full expression contained in (27). Consider

$$-\tfrac{1}{2}\int [\delta(g^{il}g^{mk}\sqrt{(-g)})\mathscr{F}^{\text{ret}}_{mli_A}]_{,k} G^{\text{ret}}_{i_B i}\,\mathrm{d}^4x. \tag{29}$$

Using (17), we replace $G^{\text{ret}}_{i_B i}$ by $G^{\text{adv}}_{i i_B}$ and apply the divergence theorem to the resulting expression. Since the metrical variations vanish on the boundary of V, (29) becomes

$$\tfrac{1}{2}\int \delta(g^{il}g^{mk}\sqrt{(-g)})\mathscr{F}^{\text{ret}}_{mli_A} G^{\text{adv}}_{ii_B,k}\,\mathrm{d}^4x.$$

Further, since $\mathscr{F}^{\text{ret}}_{mli_A} = -\mathscr{F}^{\text{ret}}_{lmi_A}$ we may rewrite (29) as

$$-\tfrac{1}{4}\int \delta(g^{il}g^{mk}\sqrt{(-g)})\mathscr{F}^{\text{ret}}_{mli_A}\mathscr{F}^{\text{adv}}_{iki_B}\,\mathrm{d}^4x.$$

The other terms can be similarly reduced and the final answer comes out as

$$
\begin{aligned}
\delta\bar{G}_{i_{\rm B}i_{\rm A}} = & -\tfrac{1}{4}\int \delta(g^{il}g^{mk}\sqrt{(-g)})(\mathscr{F}^{\rm ret}_{mli_{\rm A}}\mathscr{F}^{\rm adv}_{iki_{\rm B}}+\mathscr{F}^{\rm adv}_{mli_{\rm A}}\mathscr{F}^{\rm ret}_{iki_{\rm B}})\,{\rm d}^4x \\
& +\tfrac{1}{2}\int \delta(\sqrt{(-g)}g^{il})[(G^{\rm ret}({\rm X},{\rm A})_{,i_{\rm A}l}G^{\rm adv}_{ii_{\rm B}}+G^{\rm ret}({\rm X},{\rm B})_{,i_{\rm B}l}G^{\rm adv}_{ii_{\rm A}}) \\
& +(G^{\rm adv}({\rm X},{\rm A})_{,i_{\rm A}l}G^{\rm ret}_{ii_{\rm B}}+G^{\rm adv}({\rm X},{\rm B})_{,i_{\rm B}l}G^{\rm ret}_{ii_{\rm A}})]\,{\rm d}^4x \\
& -\tfrac{1}{2}\int \delta(\sqrt{(-g)})(G^{\rm ret}({\rm X},{\rm A})_{,i_{\rm A}}G^{\rm adv}({\rm X},{\rm B})_{,i_{\rm B}}+G^{\rm adv}({\rm X},{\rm A})_{,i_{\rm A}}G^{\rm ret}({\rm X},{\rm B})_{,i_{\rm B}})\,{\rm d}^4x.
\end{aligned}
\tag{30}
$$

That $\delta\bar{G}_{i_{\rm B}i_{\rm A}} = \delta\bar{G}_{i_{\rm A}i_{\rm B}}$ is obvious. Also, reference to figure 2 shows that $\delta\bar{G}_{i_{\rm B}i_{\rm A}} = 0$ for the case $s^2_{\rm AB} < 0$. For, in the case shown in figure 2, a typical point X of V lies in the future light cones of A and B and therefore there is no contribution from (30) to $\delta\bar{G}_{i_{\rm B}i_{\rm A}}$.

The rest of the calculation is more or less similar to that in I. Using the definitions (19) and (20), and making use of the fact that

$$
\int G^{\rm ret}({\rm X},{\rm A})_{,i_{\rm A}}\,{\rm d}a^{i_{\rm A}} = \int G^{\rm adv}({\rm X},{\rm A})_{,i_{\rm A}}\,{\rm d}a^{i_{\rm A}} = 0, \tag{31}
$$

for an integration along the entire world line of the charge a, we get

$$
\begin{aligned}
\delta\Lambda = & -4\pi\sum_{a<b}\sum e_a e_b \iint \delta\bar{G}_{i_{\rm A}i_{\rm B}}\,{\rm d}a^{i_{\rm A}}\,{\rm d}b^{i_{\rm B}} \\
= & \frac{1}{16\pi}\sum_{a<b}\sum \int \delta(\sqrt{(-g)}g^{il}g^{mk})(F^{(a)}_{ml\,\rm ret}F^{(b)}_{ik\,\rm adv}+F^{(a)}_{ml\,\rm adv}F^{(b)}_{ik\,\rm ret})\,{\rm d}^4x.
\end{aligned}
\tag{32}
$$

Comparison with (13) then gives the energy tensor as

$$
T^{ik} = \frac{1}{8\pi}\sum_{a<b}\sum (\tfrac{1}{2}g^{ik}F^{(a)mn}_{\rm ret}F^{(b)}_{mn\,\rm adv} - F^{(a)i}{}_{l\,\rm ret}F^{(b)kl\,\rm adv} - F^{(b)i}{}_{l\,\rm adv}F^{(a)kl}_{\rm ret}). \tag{33}
$$

This is the canonical tensor.

As pointed out by Wheeler and Feynman, the canonical tensor (33) differs from the Frenkel tensor (25) by a tensor of zero divergence. Hence so far as classical electrodynamics is concerned it does not matter whether we use (25) or (33). In general relativity, on the other hand, the actual values of T^{ik} and not $T^{ik}_{;k}$ are required. There the difference between (25) and (33) does matter. It is satisfactory therefore that the gravitational prescription should lead to an unambiguous tensoı.

The duality of Frenkel and canonical tensors is common in all direct particle theories. Earlier Narlikar (1968) had shown that for every tensor field of arbitrary rank described by a lagrangian bilinear in the field and its first derivatives, there exists a direct particle theory. The method discussed above can be generalized to apply to all such direct particle fields. The result will be to yield the canonical tensor as the correct stress energy tensor in all cases.

4. Response of the universe

We now return to the point raised at the end of § 2. Is the presence of advanced effects in (33) an embarrassment to the theory? To answer this question, let us first consider the situation in Maxwell field theory.

In Maxwell's theory the stress energy tensor is given by (11). In (11) we did not specify whether F_{ik} is advanced or retarded, or a mixture. Indeed such a specification is meaningless unless we relate it to a source. To fix ideas suppose we momentarily activate the charge a and consider (11) for such a situation. If we solve Maxwell's equations for the motion of a we will get a general solution of the form

$$\alpha F^{(a)}_{ik\,\mathrm{ret}} + (1-\alpha) F^{(a)}_{ik\,\mathrm{adv}}, \tag{34}$$

where α is a constant. However, in order to maintain causality we specify that there was no disturbance prior to the motion of charge a and hence choose $\alpha = 1$. Thus in this example F_{ik} in (11) is the retarded field of a.

In the theory of direct interparticle action we have to proceed differently in dealing with the same situation, and we refer to the treatment given by Hoyle and Narlikar (1963). There it was shown that in a cosmological model with a perfect future absorber but an imperfect past absorber, the following relation holds when the charge a is excited:

$$F^{(a)}_{\mathrm{ret}} - F^{(a)}_{\mathrm{adv}} + \sum_{b \neq a} (F^{(b)}_{\mathrm{ret}} - F^{(b)}_{\mathrm{adv}}) \equiv 0. \tag{35}$$

This means the field in the neighbourhood of charge a is given by

$$\tfrac{1}{2} \sum_{b} (F^{(b)}_{\mathrm{ret}} + F^{(b)}_{\mathrm{adv}}) = \sum_{b \neq a} F^{(b)}_{\mathrm{ret}} + F^{(a)}_{\mathrm{ret}} \tag{36}$$

where we have made use of (35). At first sight it would appear that (35) enables us to write (36) also as

$$\tfrac{1}{2} \sum_{b} (F^{(b)}_{\mathrm{ret}} + F^{(b)}_{\mathrm{adv}}) = \sum_{b \neq a} F^{(b)}_{\mathrm{adv}} + F^{(a)}_{\mathrm{adv}}. \tag{37}$$

However (36) and (37) have different interpretations, because of the asymmetric behaviour of the past and future absorbers. In (36), the retarded fields of $b \neq a$ arise from disturbances caused in the charges $b \neq a$ in the future absorber as a consequence of the retarded field of a, and because of attenuation by the redshift effect these are small. Certainly as (36) shows there is no disturbance *prior* to the excitation of a. (37) describes the same phenomenon in terms of advanced fields, but in this case the fields of $b \neq a$ are also large. (36) is therefore a more convenient mode of description than (37). If, on the other hand, the universe were a perfect absorber in the past and an imperfect absorber in the future the roles of (36) and (37) would be interchanged. In that case we would use (37) to describe a situation where there is no disturbance *after* a has been moved, but there is disturbance prior to the movement of a. In this case the terms $F^{(b)}_{\mathrm{adv}}$ for $b \neq a$ are small, but the terms $F^{(b)}_{\mathrm{ret}}$, $b \neq a$, are large because they arise from disturbances in the past absorber. A detailed discussion of this asymmetry is given in Hoyle and Narlikar (1963). Needless to say that of the two cases considered above only the former is consistent with the other time asymmetric processes like the expansion of the universe.

We shall accordingly use a cosmological model with a perfect future absorber and an imperfect past absorber. In such a model charges interact through retarded effects as given in (36). As in the case of Maxwell field theory we shall consider the form of the stress energy tensors (25) or (33) in the neighbourhood of charge a which has been momentarily accelerated at the world point A. To save writing too many indices, we will write a typical term of (25) or (33) simply as a product $F^{(a)} \otimes F^{(b)}$ or $F^{(a)}_{\text{ret}} \otimes F^{(b)}_{\text{adv}}$ as the case may be. Then using (35) we get

$$\sum_{b \neq a} F^{(a)}_{\text{ret}} \otimes F^{(b)}_{\text{adv}} = F^{(a)}_{\text{ret}} \otimes \sum_{b \neq a} F^{(b)}_{\text{ret}} + F^{(a)}_{\text{ret}} \otimes (F^{(a)}_{\text{ret}} - F^{(a)}_{\text{adv}}). \tag{38}$$

Of the terms on the right-hand side, the first one is negligible if there is no systematic field at A from other charges. The term $-F^{(a)}_{\text{ret}} \otimes F^{(a)}_{\text{adv}}$ is zero, since at no point other than A do the past and future light cones from A intersect. Thus only the term $F^{(a)}_{\text{ret}} \otimes F^{(a)}_{\text{ret}}$ remains. Hence in this case the canonical tensor reduces to (11) with

$$F_{ik} = \sum_{b} F^{(b)}_{\text{ret}}. \tag{39}$$

This is a 'reasonable' result.

With the Frenkel tensor we have a problem. There we have, for a typical product

$$\sum_{b \neq a} F^{(a)} \otimes F^{(b)} = F^{(a)} \otimes \sum_{b \neq a} F^{(b)}_{\text{ret}} + \tfrac{1}{2} F^{(a)} \otimes (F^{(a)}_{\text{ret}} - F^{(a)}_{\text{adv}}) \simeq \tfrac{1}{4}(F^{(a)}_{\text{ret}} \otimes F^{(a)}_{\text{ret}} - F^{(a)}_{\text{adv}} \otimes F^{(a)}_{\text{adv}}), \tag{40}$$

where the other terms have been ignored for similar reasons as those given for the canonical tensor. In this case we see that even in a completely absorbing universe, some advanced effects on geometry would persist. These effects would be embarrassingly large in the remote past of an ever expanding universe, because of blueshift. Therefore from the classical point of view the Frenkel tensor does not appear so 'reasonable'.

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